**GRADUATION THESIS**

**Tên đề tài:** Định vị trong hệ thống 5G MIMO Millimeter wave bằng phương pháp Distributed Compressive Sensing (S-OMP)

**THESIS TITLE:** Position Estimation Through MillimeterWave MIMO in 5G Systems using Distributed Compressive Sensing (S-OMP)

**ABSTRACT**

Nowadays, large antenna arrays and millimeter wave signals are thought to be key technology for upcoming 5G networks. Their potential benefits for precise positioning are largely unexplored, despite their well-known benefits for attaining high-data rate communications. In this thesis, a 5G channel using millimeter-wave (mmWave) and massive Multiple-Input Multiple-Output (mMIMO) technologies is simulated, considering the following localization parameters: Time of Arrival (TOA), Angle of Departure (AoD), and Angle of Arrival (AoA). To achieve these precise estimations, I employ an approach built upon the Distributed Compressed Sensing—Subspace Orthogonal Matching Pursuit (DCS-SOMP) algorithm. In the presence of scatterers, we estimate the Cramér-Rao bound (CRB) on location and rotation angle estimation uncertainty from millimeter wave signals from a single transmitter. Additionally, we describe a ***novel*** two-stage algorithm for position and rotation angle estimation that attains the CRB for average to high signal-to-noise ratio. For coarse estimation, the approach is based on the multiple measurement vectors matching pursuit, followed by a refinement stage based on the space alternating generalized expectation maximization (SAGE) algorithm. Finally, we estimate accurate position and rotation angle, which is possible using signals from a single transmitter, in line-of-sight, non-line-of-sight, or obstructed-line-of-sight scenarios.

***Keywords: :*** *5G; Distributed* *compressed sensing; DCS-SOMP; parameter estimation; position estimation; mmWave; mMIMO*

**TÓM TẮT**

***Từ khóa:*** *5G; Distributed* *compressed sensing; DCS-SOMP; parameter estimation; position estimation; mmWave; mMIMO*

**AUTHORSHIP**

*“I hereby declare that the work contained in this thesis is of my own and has not been previously submitted for a degree or diploma at this or any other higher education institution. To the best of my knowledge and belief, the thesis contains no materials previously published or written by another person except where due reference or acknowledgement is made.”*

Signature:………………………………………………

**SUPERVISOR’S APPROVAL**

*“I hereby approve that the thesis in its current form is ready for committee examination as a requirement for the Bachelor of Electronics and Telecommunication degree at the University of Engineering and Technology.”*

Signature:………………………………………………

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I am grateful to … (should be your tutor)

I would like to also thank … (should be your colleagues, friends who have helped you along)

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**ABBREVIATIONS**

**CHAPTER 1: INTRODUCTION**

*(Tính cần thiết của đề tài, ý nghĩa khoa học và thực tiễn, đối tượng và phương pháp nghiên cứu, nội dung nghiên cứu)*

**1.1. Motivation**

In today's technological age, the application of 5G networks is growing in popularity, and having the ability to accurately estimate the location and angle of rotation of devices in 5G networks will play an important role in many areas such as intelligent transportation, object tracking and positioning, and personal communications

Mm-wave and massive multiple-input-multiple-output (MIMO) will likely be adopted in fifth generation (5G) communication networks, thanks to a number of favorable properties. Particularly, due to exploiting the carrier frequencies beyond 30 GHz and large available bandwidth, mm-wave can provide high data rate. This can be obtained through dense spatial multiplexing with large antennas. A sparse signal recovery problem exploiting the sparse nature of mm-wave channels is formulated for channel estimation based on the parametric channel model with quantized angles of departures/arrivals (AoDs/AoAs), called the angle grids. The problem is solved by the orthogonal matching pursuit (OMP) algorithm employing a redundant dictionary consisting of array response vectors with finely quantized angle grids. However, OMP (Orthogonal matching pursuit) is only used for single subcarriers, to estimate accurate position and rotation angle, S-OMP Algorithm for multiple subcarriers (Simultaneous orthogonal matching pursuit) is used. Due to the linear antenna array, the method applies to a 2D environment. Additionally, the DCS-SOMP method provides only a coarse parameter estimate, demanding further fine-tuning using the SAGE method.

**1.2. Related work**

Channel estimation in mobile communication systems is very necessary. Channel estimation aims to reduce the variance of the function transmission of the transmit channel compared to the receive channel due to many reasons transmission process. Channel estimation can be performed in different ways: with or without the support of parametric modeling, using correlation Observe the frequency or time of the radio channel, based on the blind or pilot (training), adaptive or non-adaptive. Among them, channel estimation using compressed sensing is one of the most popular methods. In [1] presents the method to estimate accurate position and rotation angle estimation is possible using signals from a single transmitter, in either line-of-sight, non-line-of-sight, or obstructed-line-of-sight conditions. Knowledge about Distributed Compressive Sensing and Joint Sparsity Modles is discussed in two studies [2] and [7]. In [4], author model a 5G downlink channel using millimeter-wave (mmWave) and massive Multiple-Input Multiple-Output (mMIMO) technologies, considering the following localization parameters: Time of Arrival (TOA), Two-Dimensional Angle of Departure (2D-AoD), and Two-Dimensional Angle of Arrival (2D-AoA), both encompassing azimuth and elevation. In [5], the classic method to solve problem optimizing L1 norm (direct L1) is alternating minimization (AM) via proximal gradient descent. A joint heuristic beam selection and user position and orientation tracking approach is proposed in [6].

The OMP algorithm and variations of the OMP algorithm using for channel estimation are also studied in the articles below. In paper [8] demonstrates theoretically and empirically that a greedy algorithm called Orthogonal Matching Pursuit (OMP). Paper [11] seeks to bridge the two major algorithmic approaches to sparse signal recovery from an incomplete set of linear measurements – L1-minimization methods and iterative methods (Matching Pursuits) and a simple regularized version of Orthogonal Matching Pursuit (ROMP). Stagewise Orthogonal Matching Pursuit algorithm is proposed in [12]. Channel estimation provides information of the AOA/AOD and thus of the relative location of the transmitter and receiver. In [9], author propose an efficient open-loop channel estimator for a millimeter-wave (mm-wave) hybrid multiple-input multiple-output (MIMO) system consisting of radio-frequency (RF) beamformers with large antenna arrays followed by a baseband MIMO processor. Article [10] reconsider the role of NLOS components for position and orientation estimation in 5G millimeter wave MIMO systems and is based on the concept of Fisher information

**1.3. Contributions and thesis overview**

The contributions of this thesis are described as follows:

- This thesis presents a method for estimating position and angle of rotation accurately through mm-wave signals from a single transmitter, even in conditions of obstructions. - This method achieves the Cramér-Rao limit (CRB) for the estimation of position and angle of rotation under the signal-from-one-way-mains-correct condition from a single transmitter.

- The method proposed in the thesis uses advanced signal processing and measurement techniques such as compressed sensing and expectation maximization algorithms to achieve accurate position and angle estimation. This method is different and advanced from traditional methods. This opens up the potential of mm-wave signals and large MIMO antennas in locating and orienting devices in 5G networks.

- This thesis proposes a method for determining position and direction using mm-wave signals from a single emitter, including in conditions of obstacles.

- The results of the study show that it is possible to determine the correct position and direction using magnetic signals from a single emitter, regardless of whether or not a direct line of sight, an indirect line of sight, or an obscured line of sight.

**1.4. Thesis layout**

The remainder of this article is organized as follows:

In Chapter 2, a literature review about basic theories of 5G system including system model, basic theory of compressed sensing and methods for 5G mm-wave channel estimation is presented.

Chapter 3 presented the details of positioning problem through millimeter wave MIMO in a 5G systemincluding overview about channel estimation, OMP Algorithm, S-OMP Algorithm and positioning methods using channel information (channel estimation).

In Chapter 4, simulation results are presented and discussed.

**CHAPTER 2: BASIC THEORIES OF 5G SYSTEM**

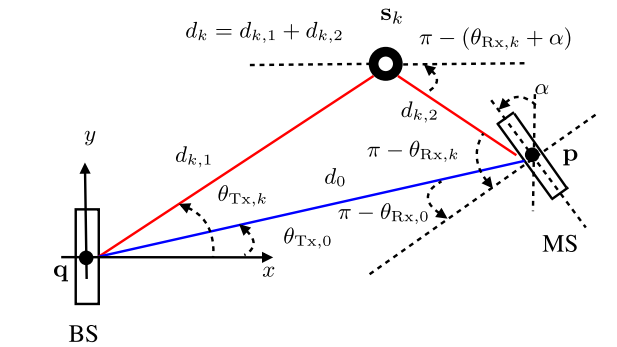
**2.1. System Model**

A MIMO system with a BS equipped with antennas and a MS equipped by antennas operating at a carrier frequency (corresponding to wavelength ) and bandwidth B. Locations of the BS and MS are denoted by and with the α ∈ [0, 2π) denoting the rotation angle of the MS’s antenna array. While *p* and *α* are unknown, the value of *q* is assumed to be known.

*2.1.1. Transmitter Model*

The transmission of orthogonal frequency division multiplexing (OFDM) signals, where a BS with hybrid analog/digital precoder communicates with a single MS. At the BS, *G* signals are transmitted sequentially, where the *g*-th transmission comprises simultaneously transmitted symbols for each subcarrier . The symbols are first precoded and then transformed to the time-domain using Npoint inverse fast Fourier transform (IFFT). A cyclic prefix (CP) of length is added before applying the radio-frequency (RF) precoding where D is the length of CP in symbols. Here, denotes the sampling period and is assumed to exceed the delay spread of the channel. The transmitted signal over subcarrier n at time g can be expressed as . The beamforming matrix is defined as where is implemented using the analog phase shifters with the entries of the form , where are given phases, and is the digital beamformer, and overall they satisfy a total power constraint . Considering the sparsity of the mm-wave channels one usually needs much less beams than antenna elements , i.e., . Also, the presence of in the proposed model leads to the extension of system model to multi-user mm-wave downlink systems with a limited feedback channel from MSs to the BS. A general expressions that permit the study of the impact on performance and optimization of different choices of beamformers and signals , although this is out of the scope of the paper. My approach is also compatible with beam reference signal (initial access) procedures, and it could be complemented with a Bayesian recursive tracker with user-specific precoding.

*2.1.2. Channel Model*



*Figure 1*: Two dimensional illustration of the LOS (blue link) and NLOS (red link) based positioning problem. The BS location ***q*** and BS orientation are known, but arbitrary. The location of the MS ***p***, scatterer , rotation angle ***α,*** *AOAs(, AODs(),* the channel between BS and MS, and scatterers and the distance between the antenna centers are unknown

Fig. 1 shows the position-related parameters of the channel. These parameters include , and , , denoting the AOA, AOD, and the path length (with time-of-arrival (TOA) and the speed of light ) of the k-th path (k = 0 for the LOS path and k > 0 the NLOS paths). For each NLOS path, there is a scatterer with unknown location , for which we define and

The channel model is introduced, under a frequency-dependent array response, suitable for wideband communication (with fractional bandwidth up to 50%). Assuming K +1 paths and a channel that remains constant during the transmission of G symbols, the channel matrix associated with subcarrier n is expressed as

for response vectors

,

,

and

for path loss and complex channel gain , respectively, of the k-th path. For later use, and

The structure of the frequency-dependent antenna steering and response vectors and depends on the specific array structure. For the case of a uniform linear array (ULA), which will be the example studied in this thesis, (the response vector is obtained similarly)

where is the signal wavelength at the *n*-th subcarrier and d denotes the distance between the antenna elements (we will use ). When , , and reverts to the standard narrow-band model.

*2.1.3. Received Signal Model*

The received signal for subcarrier n and transmission , after CP removal and fast Fourier transform (FFT), can be expressed as

where is a Gaussian noise vector with zero mean and variance per real dimension.

Now, the goal is estimating the position **p** and orientation α of the MS from .

**2.2. Basic theory of compressed sensing**

Compressed sensing is an emerging field based on the revelation that a small collection of linear projections of a sparse signal contains enough information for reconstruction. A new framework for single-signal sensing and compression has developed recently under the rubric of Compressed Sensing (CS) [2, 3]. CS builds on the surprising revelation that a signal having a sparse representation in one basis can be recovered from a small number of projections onto a second basis that is incoherent with the first. (Roughly speaking, incoherence means that no element of one basis has a sparse representation in terms of the other basis). In fact, for an N-sample signal that is K-sparse (By K-sparse, we mean that the signal can be written as a sum of K basis functions.), roughly *cK* projections of the signal onto the incoherent basis are required to reconstruct the signal with high probability (typically c ≈ 3). This has promising implications for applications involving sparse signal acquisition. Instead of sampling a K-sparse signal N times, only *cK* incoherent measurements suffice, where K can be orders of magnitude less than N. Moreover, the *cK* measurements need not be manipulated in any way before being transmitted, except possibly for some quantization. Interestingly, independent and identically distributed Gaussian or Rademacher (random ±1) vectors provide a useful universal measurement basis that is incoherent with any given basis with high probability.

Suppose that *x* is a signal, and let be a basis or *dictionary* of vectors. When we say that *x* is sparse, we mean that *x* is well approximated by a linear combination of a small set of vectors from . That is, where ; we say that *x* is *K-sparse* in *Ψ* and call *Ψ* the sparse basis. The CS theory states that it is possible to construct an M × N *measurement* matrix Φ, where , yet the measurements *y =* Φ*x* preserve the essential information about *x*. For example, let Φ be a *cK × N* random matrix with independently and identically distributed. Gaussian entries, where *c = c(N, K)* is an *oversampling factor*. Using such a matrix it is possible, with high probability, to recover any signal that is *K*-sparse in the basis Ψ from its image under Φ. For signals that are not *K*-sparse but *compressible*, meaning that their coefficient magnitudes decay exponentially, there are tractable algorithms that achieve not more than a multiple of the error of the best *K*-term approximation of the signal.

Several algorithms have been proposed for recovering *x* from the measurements *y*, each requiring a slightly different constant *c*. The canonical approach uses linear programming to solve the minimization problem.

subject to Φ*Ψθ* = *y*.

This problem requires but has somewhat high computational complexity. Additional methods have been proposed involving greedy pursuit methods. Examples include Matching Pursuit (MP) and Orthogonal Matching Pursuit (OMP), which tend to require fewer computations but at the expense of slightly more measurements [5].

**2.3. Methods for 5G mm-wave channel estimation**

*2.3.1. Channel estimation using sparse CS methods*

2.3.1.1. L1 norm minimization (L1 trực tiếp)

Because is a sparse vector, it cannot be solved using conventional Least Square (LS). To solve the above problem, add a quantity to the loss function that helps control the sparsity in the vector. The problem is transformed into an equivalent problem:

Using L1 standard minimization as follows:

The method to solve this problem is alternating minimization (AM) through proximal gradient descent. The main idea of the AM method is to optimize a group of expressions and use the results just found to optimize the rest through proxima gradient descent. The process is repeated until the value of the loss function is reduced to a certain threshold. To ensure there is decay at each loop, updated according to the derivative of the first expression cluster as follows:

)

Then, is optimized according to the second expression cluster based on the results just found in the first step:

The optimization process is rewritten as

The solution to the problem is as follow:

2.3.1.2. Tối thiểu tổng các giá trị suy biến (L1 gián tiếp)

The proposed idea is based on a number of existing techniques, including: Iterative ShrinkageThresholding Algorithm [8], Weighted Nuclear Norm Minimization (WNNN) [9] and Greedy Efficient Component Optimization (GECO) [11]. Apply method Lagrangian method for (15), our problem becomes zero optimal forced as follows:

Some studies suggest that larger degeneracy values have a large influence rather than approximating a matrix, so one often multiplies on each value degenerate a weight to obtain a more accurate estimate, above problem becomes:

where is the corresponding weight, R is the rank of the matrix, and µ ∈ is the parameter used for adjustment. The first quantity is a convex function and the second quantity is a function not continuously. Following the approach can minimize by to limit the first expression cluster by

If the second expression cluster is a convex function, for example , can decompose the loss function and then optimize it element by element as before presented in Section 2.3.1.1. However, cannot apply this method because the second expression cluster has a one-to-many relationship with the elements of . The idea is still AM, can still optimize the first cluster and use it the results just found to optimize the second cluster but in a different way. The first cluster gives us an update on as follows:

The next step is to use the rank constraint of the matrix to optimize the clustering two. To do this, after obtaining in the first step, can arrange the vector into matrix form and use of properties

=

where r is rank of matrix

(\*)

if the weights are sorted in a zero order reduced, problem can be evaluated in closed form:

where is the degeneracy analysis of and (·) is the decision-making threshold comes with its weight ***w***

where is estimated by

where is constant and .

However, this is a method applied to the problem of image denoising. It can be seen that the image processing problem is only concerned with minimizing the sum of degenerate values and has no constraints on the rank of the image matrix. Therefore, the solution to problem (\*) is not necessarily the solution to the channel estimation problem because the sum of small degenerate values does not guarantee that the rank of the matrix is small and vice versa. Unlike image processing, the problem of sparse channel estimation is also concerned with the rank of the channel matrix (fixed-rank constraint), with the rank of the channel matrix representing the number of dominant links. Therefore, problem (\*) needs additional constraints as follows:

s.t rank (

Because the matrix rank constraint makes the problem non-convex, the idea is to introduce constraints into the search space and solve the problem of unconstrained optimization on a constrained search space. Here, we consider [14] as a reference. Embedding all or part of the algorithm steps in [14] into algorithm [13] to find the optimal solution. Line 7 of the algorithm in [13] performs degeneracy analysis (SVD) for the image matrix (in this problem, the channel matrix), the purpose is to extract degeneracy values and then estimate using Use Line 8. This does not make the channel/image matrix sparse, so embedding the constraint rank (. When introduce the matrix rank constraint into the problem, another matrix approximation problem using matrices with rank equal to 1 is appeared. After obtaining the current estimated channel matrix, perform optimization according to [14]

*2.3.2. Sparse Bayesian Inference*

**Tổng kết chương II**

**CHAPTER 3: POSITIONING PROBLEM THROUGH MILLIMETER WAVE MIMO IN 5G SYSTEM**

**3.1. Overview about channel estimation**

**3.2. Distributed Compressive Sensing – Joint Sparsity Modles (JSM)**

*3.2.1. Theory for DCS*

In this thesis, theory and algorithms for *distributed compressed sensing* (DCS) that exploit both intra- and inter-signal correlation structures is presented. In a typical DCS scenario, a number of sensors measure signals (of any dimension) that are each individually sparse in some basis and also correlated from sensor to sensor. Each sensor *independently* encodes its signal by projecting it onto another, incoherent basis (such as a random one) and then transmits just a few of the resulting coefficients to a collection point. Under the right conditions, a decoder at the collection point can jointly reconstruct all of the signals precisely.

The DCS theory rests on a concept that we term the *joint sparsity* of a signal ensemble. The first model for jointly sparse signals and proposed corresponding joint reconstruction algorithms is introduced. Derived results on the required measurement rates for signals that have sparse representations under each of the models: while the sensors operate entirely *without collaboration*, dramatic savings relative to the number measurements required for separate CS decoding.

*3.2.2. Joint Sparsity Modles (JSM)*

In this section, the notion of a signal being sparse in some basis to the notion of an ensemble of signals being jointly sparse is generalized. Two different joint sparsity models (JSMs) that apply in different situations is considered. In these models, each signal is itself sparse, and the CS framework from above to encode and decode each one separately is used. However, there also exists a framework wherein a joint representation for the ensemble uses fewer total vectors.

The following notation for signal ensembles and our measurement model is used. Denote the signals in the ensemble by , j ∈ {1, 2, . . . , J}, and assume that each signal ∈ . (n) to denote sample *n* in signal j, and assume that there exists a known sparse basis Ψ for in which the can be sparsely represented. The coefficients of this sparse representation can take arbitrary real values (both positive and negative). Denote by the *measurement matrix* for signal *j*; is ×N and, in general, the entries of are different for each j. Thus, = consists of < N incoherent measurements of , emphasize random i.i.d. Gaussian matrices in the following, but other schemes are possible, including random ±1 Bernoulli/Rademacher matrices.

3.2.2.1. JSM-1: Sparse common component + innovations

In this model, all signals share a common sparse component while each individual signal contains a sparse innovation component; that is,

= z + , j ∈ {1, 2, . . . , J}

with z = Ψ, = K and = Ψ, = . Thus, the signal *z* is common to all of the and has sparsity K in basis Ψ. The signals are the unique portions of the and have sparsity in the same basis. A practical situation well-modeled by JSM-1 is a group of sensors measuring temperatures at several outdoor locations throughout the day. The temperature readings have both temporal (intra-signal) and spatial (intersignal) correlations. Global factors, such as the sun and prevailing winds, could have an effect *z* that is both common to all sensors and structured enough to permit sparse representation. More local factors, such as shade, water, or animals, could contribute localized innovations that are also structured (and hence sparse). A similar scenario could be imagined for a network of sensors recording light intensities, air pressure, or other phenomena. All these scenarios correspond to measuring properties of physical processes that change smoothly in time and in space and thus are highly correlated.

3.2.2.2. JSM-2: Common spare supports model

In this model all signals are constructed from the same sparse set of basis vectors, but with different coefficients:

= Ψ, *j ∈ {1, 2, . . ., J}*

where each is supported only on the same Ω ⊂ {1, 2, . . ., N} with |Ω| = K. Hence, all signals are *K*-sparse and are constructed from the same *K* elements of Ψ, but with arbitrarily different coefficients. A practical situation well-modeled by JSM-2 is where multiple sensors acquire the same signal but with phase shifts and attenuations caused by signal propagation. In many cases it is critical to recover each one of the sensed signals, such as in many acoustic localization and array processing algorithms. Another useful application for JSM-2 is MIMO communication [9].

*3.2.3. Reconstruction Algorithms*

3.2.3.1. Recovery via One-Step Greedy Algorithm (OSGA)

When there are many correlated signals in the ensemble, a simple non-iterative greedy algorithm based on inner products will suffice to recover the signals jointly. For simplicity but without loss of generality, we assume that Ψ = (it can be absorbed by the measurement matrix) and that an equal number of measurements = M are taken of each signal. We write in terms of its columns: = [, , . . . , ]. The algorithm follows:

**1) Get greedy**: Given all of the measurements, compute the test statistics

for n ∈ {1, 2, . . . , N} and estimate the common coefficient support set by Ω = {n having one of the K largest }. When the sparse, nonzero coefficients are sufficiently generic (as defined below), we have the following surprising result.

*Theorem 1*: Let Ψ be an orthonormal basis for , let the measurement matrices contain i.i.d. Gaussian entries, and assume that the nonzero coefficients in the are i.i.d. Gaussian random variables. Then with M ≥ 1 measurements per signal, OSGA recovers Ω with probability approaching one as J → ∞. In words, with fewer than K measurements per sensor, it is possible to recover the sparse support set Ω under the JSM-2 model. Of course, this approach does not recover the K coefficient values for each signal; that requires K measurements per sensor.

*Theorem 2*: Assume that the nonzero coefficients in the are i.i.d. Gaussian random variables. Then the following statements hold:

1) Let the measurement matrices contain i.i.d. Gaussian entries, with each matrix having an oversampling factor of c = 1 (that is, = K for each measurement matrix ). Then OSGA recovers all signals from the ensemble {} with probability approaching one as J → ∞.

2) Let be a measurement matrix with oversampling factor c < 1 (that is, < K), for some j ∈ {1, 2, . . . , J}. Then with probability one, the signal xj cannot be uniquely recovered by any algorithm for any value of J.

The first statement is an immediate corollary of Theorem 1; the second statement follows because each equation would be underdetermined even if the nonzero indices were known. Thus, under the JSM-2 model, the one-step greedy algorithm asymptotically performs as well as an oracle decoder that has prior knowledge of the locations of the sparse coefficients.

Theorem 2 provides tight achievable and converse bounds for JSM-2 signals, in the sense that the number of measurements needed for success is only one greater than the number that yields reconstruction failure. OSGA works well even when M is small, as long as J is sufficiently large. However, in the case of fewer signals (small J), OSGA performs poorly. An alternative recovery technique based on simultaneous greedy pursuit proposed in next part that performs well for small J.

3.2.3.2. Recovery via iterative greedy pursuit

In practice, the common sparse support among the J signals enables a fast iterative algorithm to recover all of the signals jointly, Simultaneous Orthogonal Matching Pursuit (SOMP) [6], which can be readily applied in our DCS framework. SOMP is a variant of OMP that seeks to identify Ω one element at a time. The DCS-tailored is dubed SOMP algorithm DCS-SOMP. To adapt the original SOMP algorithm to our setting, first extend it to cover a different measurement basis for each signal . Then, in each DCS-SOMP iteration, the column index n ∈ {1, 2, . . . , N} is selected that accounts for the greatest amount of residual energy across all signals. As in SOMP, the remaining columns (in each measurement basis) is orthogonalizeed after each step; after convergence we obtain an expansion of the measurement vector on an orthogonalized subset of the holographic basis vectors. To obtain the expansion coefficients in the sparse basis, then the orthogonalization process is reversed using the QR matrix factorization. The algorithm is as follows

1) Initialize: Set the iteration counter ℓ = 1. For each signal index *j ∈ {1, 2, . . . , J*}, initialize the orthogonalized coefficient vectors = 0, ∈ ; also initialize the set of selected indices Ω = ∅. Let denote the residual of the measurement remaining after the first ℓ iterations, and initialize = .

(2) Select: the dictionary vector that maximizes the value of the sum of the magnitudes of the projections of the residual, and add its index to the set of selected indices

3) Orthogonalize: the selected basis vector against the orthogonalized set of previously selected dictionary vectors

4) Iterate: Update the estimate of the coefficients for the selected vector and residuals

5) Check for convergence: If > for all j, then increment ℓ and go to Step 2; otherwise, continue to Step 6. The parameter determines the target error power level allowed for algorithm convergence. Note that due to Step 3 the algorithm can only run for up to M iterations.

6) De-orthogonalize: Apply QR factorization on the mutilated basis = = . Since , where is the mutilated coefficient vector, we can compute the signal estimates {} as , , where is the mutilated version of the sparse coefficient vector .

In practice, each sensor projects its signal via to produce cK measurements for some c. The decoder then applies DCS-SOMP to reconstruct the J signals jointly. We orthogonalize because as the number of iterations approaches M the norms of the residues of an orthogonal pursuit decrease faster than for a non-orthogonal pursuit.

Thanks to the common sparsity structure among the signals, we believe that DCS-SOMP will succeed with < c(S). Empirically, we have observed that a small number of measurements proportional to K suffices for a moderate number of sensors J. We conjecture that K + 1 measurements per sensor suffice as J → ∞. Thus, in practice, this efficient greedy algorithm enables an oversampling factor = (K + 1)/K that approaches 1 as J, K, and N increase.

**3.3. OMP Algorithm**

OMP (Orthogonal matching pursuit) - single subcarrier

*3.3.1. Sparse recovery problems*

Sparse recovery problems arise in many applications ranging from medical imaging to error correction. Suppose *v* is an unknown *d*-dimensional signal with at most *n ≪ d* nonzero components: v ∈ , |supp(*v*)| ≤ *n* ≪ *d,* such signals n-sparse.

Suppose collect *N ≪ d* nonadaptive linear measurements of *v*, and wish to efficiently recover *v* from these. The measurements are given as the vector ∈ , where Φ is some *N × d* measurement matrix.

Exact recovery is possible with just *N = 2n*. However, recovery using only this property is not numerically feasible; the sparse recovery problem in general is known to be NP-hard. Nevertheless, massive recent work in the emerging area of Compressed Sensing demonstrated that for several natural classes of measurement matrices *Φ*, the signal *v* can be exactly reconstructed from its measurements with . In other words, the number of measurements *N ≪ d* should be almost linear in the sparsity *n*. The two major algorithmic approaches to sparse recovery are methods based on L1-minimization and iterative methods (Matching Pursuits).

*3.3.2. Orthogonal Matching Pursuit*

A popular approach to sparse recovery is via iterative algorithms, which find the support of the *n*-sparse signal *v* progressively. Once *S* = supp(*v*) is found correctly, it is easy to compute the signal *v* from its measurements *x* = as , where denotes the measurement matrix Φ restricted to columns indexed by *S*.

A basic iterative algorithm is Orthogonal Matching Pursuit (OMP). OMP recovers the support of *v*, one index at a time, in *n* steps. Under a hypothetical assumption that *Φ* is an isometry, i.e. the columns of *Φ* are orthonormal, the signal v can be exactly recovered from its measurements *x = Φv* as *v = Φ∗x*.

The problem is that the *N × d* matrix *Φ* is never an isometry in the interesting range where the number of measurements *N* is smaller than the ambient dimension *d*. Even though the matrix is not an isometry, one can still use the notion of coherence in recovery of sparse signals. In that setting, greedy algorithms are used with incoherent dictionaries to recover such signals. In setting, for random matrices one expects the columns to be approximately orthogonal, and the observation vector *u = Φ∗x* to be a good approximation to the original signal *v*.

The biggest coordinate of the observation vector *u* in magnitude should thus be a nonzero coordinate of the signal v, then find one point of the support of *v*.

Then OMP can be described as follows. First, initialize the residual *r = x*. At each iteration, compute the observation vector *u = Φ∗ r*. Denoting by *I* the coordinates selected so far, solve a least squares problem and update the residual ; , to remove any contribution of the coordinates in *I*. OMP then iterates this procedure n times, and outputs a set *I* of size *n*, which should equal the support of the signal *v*. The performance of OMP for Gaussian measure- ment matrices *Φ*; a similar result holds for general subgaussian matrices. For every fixed *n*-sparse *d*-dimensional signal *v*, and an *N × d* random Gaussian measurement matrix *Φ*, OMP recovers (the support of) v from the measurements *x = Φv* correctly with high probability, provided the number of measurements is *N ∼ n log d*.

*3.3.3. Advantages and challenges of OMP*

Orthogonal Matching Pursuit is quite fast, both theoretically and experimentally. It makes *n* iterations, where each iteration amounts to a multiplication by a *d × N* matrix (computing the observation vector *u*), and solving a least squares problem in dimensions at most *N × n* (with matrix ). This yields strongly polynomial running time. In practice, OMP is observed to perform faster and is easier to implement than L1-minimization.

Orthogonal Matching Pursuit is quite transparent: at each iteration, it selects a new coordinate from the support of the signal v in a very specific and natural way. In contrast, the known L1-minimization solvers, such as the simplex method and interior point methods, compute a path toward the solution. However, the geometry of L1 is clear, whereas the analysis of greedy algorithms can be difficult simply because they are iterative.

On the other hand, Orthogonal Matching Pursuit has weaker guarantees of exact recovery. Unlike L1-minimization, the guarantees of OMP are non-uniform: for each fixed sparse signal v and not for all signals, the algorithm performs correctly with high probability. Rauhut has shown that uniform guarantees for OMP are impossible for natural random measurement matrices.

Moreover, OMP’s condition on measurement matrices is more restrictive than the Restricted Isometry Condition. In particular, it is not known whether OMP succeeds in the important class of partial Fourier measurement matrices.

In next part, a new algorithm for sparse recovery will be introduced as the modification of Orthogonal Matching Pursuit.

***3.4. S-OMP Algorithm***

*3.4.1. S-OMP (Simultaneous orthogonal matching pursuit) - multiple subcarrier*

SOMP is a variant of OMP that seeks to identify Ω one element at a time. A new approach to this problem is presented here: a greedy pursuit algorithm called Simultaneous Orthogonal Matching Pursuit. The algorithm calculates simultaneous approximations whose error is within a constant factor of the optimal simultaneous approximation error.

A dictionary *D* is a finite collection of unit-norm elementary signals, called atoms, that spans the signal space. Each atom is denoted , where ω is drawn from an index set *Ω*. The number of atoms *N* is typically much larger than the dimension *d* of the signal space. We also define the *d × N* dictionary matrix *Φ* whose columns are atoms.

Suppose that S is a *d × K* matrix whose columns are input signals. Goal is to approximate all K input signals using different linear combinations of the same *T* atoms. Typically, *T* is much smaller than the dimension of the signal space, so the approximation is sparse. More precisely, the *simultaneous sparse approximation problem* (SSA) elicits an *N ×K* coefficient matrix *C* that solves the mathematical program min

subject to the matrix *C* has at most T nonzero rows. (SSA)

The squared Frobenius matrix norm returns the sum of the squares of the entries in a matrix.

The (SSA) problem arises if given multiple observations of a sparse input signal that are contaminated with noise. For example, the k-th input signal might have the form where νk is a realization of some random process and where x can be expressed using a linear combination of T atoms. The goal is to identify the atoms that comprise x.

To solve (SSA), a greedy pursuit method, Simultaneous Orthogonal Matching Pursuit (S-OMP) is proposed. For general dictionaries, (SSA) cannot be solved without checking every combination of T nonzero rows. Nevertheless, S-OMP correctly solves the simultaneous sparse approximation problem, provided that the atoms are weakly correlated. To quantify this property, define the coherence parameter of the dictionary,

.

When the coherence parameter is small, each pair of atoms is nearly orthogonal. Any algorithm can obtain provably good solutions to (SSA). A simple version of our result follows. Suppose that the set Λopt indexes the T atoms that appear in some solution to (SSA). Then define the *d × K* matrix whose k-th column is the best approximation of the k-th input signal using the T atoms listed in .

The algorithm S-OMP performs much better in practice than theory predicts. Not only can recover the input signals when *T* is large, the error does not grow as quickly as bounds would suggest. Moderate noise levels create surprising difficulties for algorithm.

*3.4.2. Formal description of the algorithm.*

**Algorithm 2 (S-OMP)**

- INPUT:

• A d × K matrix S of input signals

• The number T of atoms in the approximation

- OUTPUT:

• A set ΛT containing T indices

• A d × K approximation matrix

• A d × K residual matrix

- PROCEDURE:

1. Initialize the residual matrix = S, the index set = ∅, and the iteration counter t = 1.

2. Find an index that solves the easy optimization problem

denote the *k*-th canonical basis vector.

3. Set

4. Determine the orthogonal projector onto span of the atoms indexed in .

5. Calculate the new approximation and residual: and

6. Increment *t*, and return to Step 2 if *t ≤ T*

This procedure reduces to standard Orthogonal Matching Pursuit when K = 1.

Step 2 of the algorithm is referred to as the greedy selection. The intuition behind maximizing the sum of absolute correlations is that need to find an atom that contributes the most energy to as many of the input signals as possible. This absolute sum can also be written as.

Steps 4 and 5 have been written to emphasize the conceptual structure of the algorithm. It is possible to implement them much more efficiently using standard techniques for least-squares problems. Each column of the residual is orthogonal to the atoms indexed in . Therefore, no atom is ever chosen twice

* + 1. *Advantages of S-OMP compared to OMP*
    2. *Application S-OMP into DSP*

In practice, the common sparse support among the *J* signals enables a fast iterative algorithm to recover all of the signals jointly. Simultaneous Orthogonal Matching Pursuit (SOMP) can be readily applied in our DCS framework. SOMP is a variant of OMP that seeks to identify Ω one element at a time. The DCS-tailored SOMP algorithm DCS-SOMP is dubed.

To adapt the original S-OMP algorithm to setting, first extend it to cover a different measurement basis for each signal . Then, in each DCS-SOMP iteration, we select the column index n ∈ {1, 2, . . ., N} that accounts for the greatest amount of residual energy across all signals. As in SOMP, we orthogonalize the remaining columns (in each measurement basis) after each step; after convergence we obtain an expansion of the measurement vector on an orthogonalized subset of the holographic basis vectors. To obtain the expansion coefficients in the sparse basis, we then reverse the orthogonalization process using the QR matrix factorization. The algorithm is as follows:

* 1. ***Positioning methods using channel information (channel estimation)***

Channel estimation provides information of the AOA/AOD and thus of the relative location of the transmitter and receiver

Tổng kết chương III

**CHAPTER 4: SIMULATION**

4.1. Simulation Setup

4.2. Simulation Results

4.3. Discussion

Tổng kết chương IV

**CONCLUSION**

Conclusions

Future Works

**APPENDIX**

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